How to Write Fast Numerical Code

Spring 2016
Lecture: Roofline model

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Operational Intensity Again

- **Definition: Given a program P, assume cold (empty) cache**

  Operational intensity: $I(n) = \dfrac{W(n)}{Q(n)}$

  — #flops (input size n)

  — #bytes transferred cache $\leftrightarrow$ memory (for input size n)

- **Examples: Determine asymptotic bounds on I(n)**
  - Vector sum: y = x + y   **O(1)**
  - Matrix-vector product: y = Ax   **O(1)**
  - Fast Fourier transform   **O(log(n))**
  - Matrix-matrix product: C = AB + C   **O(n)**

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Compute/Memory Bound

- **A function/piece of code is:**
    - *Compute bound* if it has high operational intensity
    - *Memory bound* if it has low operational intensity

- **The roofline model makes this more precise**
- **Blackboard**

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Roofline Measurements

- **Tool developed in our group**
  (G. Ofenbeck, R. Steinmann, V. Caparros-Cabezas, D. Spampinato)
  http://www.spiral.net/software/roofline.html
- **You can use it in your project**
- **Example plots follow**
- **Get (non-asymptotic) bounds on I:**
    - daxpy: y = αx+y
    - dgemv: y = Ax + y
    - dgemm: C = AB + C
    - FFT

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Roofline Measurements

Core i7 Sandy Bridge, 6 cores
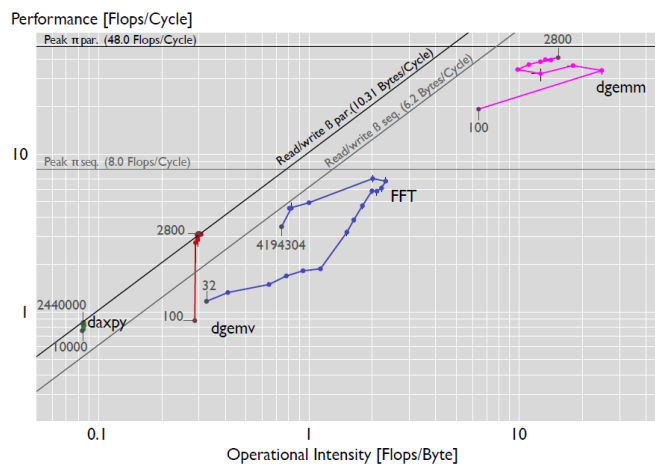*Code: Intel MKL, sequential*
Cold cache
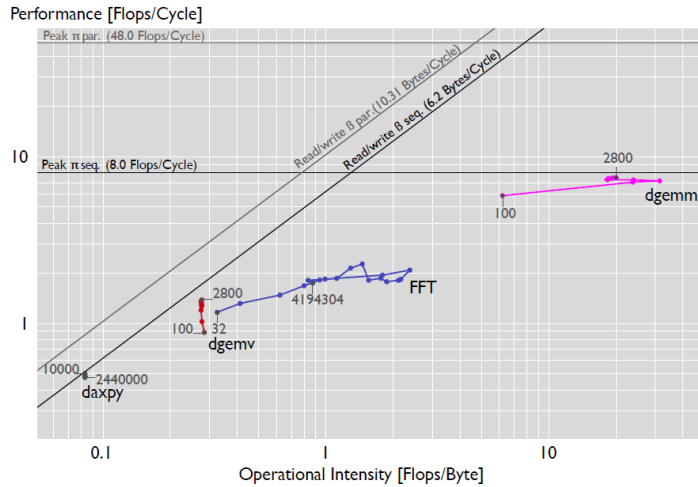
What happens when we go to parallel code?

Roofline Measurements

Core i7 Sandy Bridge, 6 cores
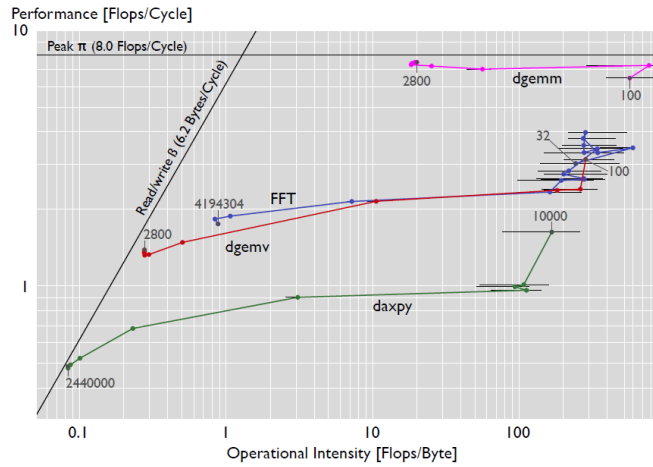*Code: Intel MKL, parallel*
Cold cache

What happens when we go to warm cache?

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How to write fast numerical code
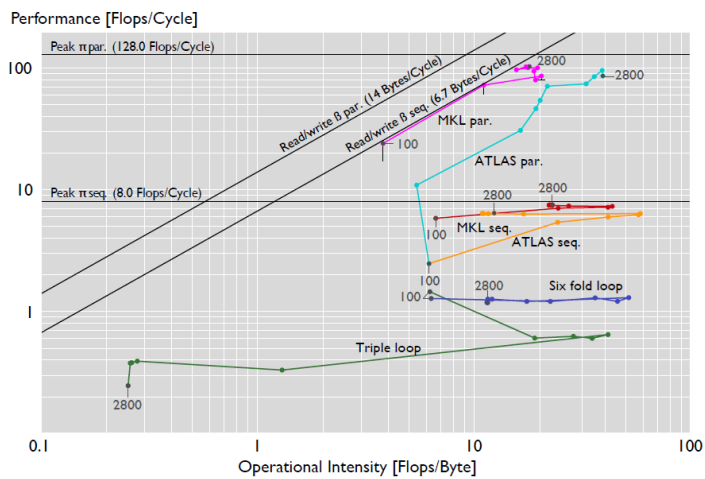*Spring 2016*

Roofline Measurements

Core i7 Sandy Bridge, 6 cores
Code: Intel MKL, *sequential*
Warm cache

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Roofline Measurements

Core i7 Sandy Bridge, 6 cores
Code: Various MMM
Cold cache

MMM: Try to guess the basic shapes

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Summary

- **Roofline plots distinguish between memory and compute bound**
- **Can be used on paper**
- **Measurements difficult (performance counters) but doable**
- **Interesting insights:** *use in your project!*

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