

How to Write Fast Numerical Code

Spring 2016

Lecture: Benchmarking, Compiler Limitations

Instructor: Markus Püschel

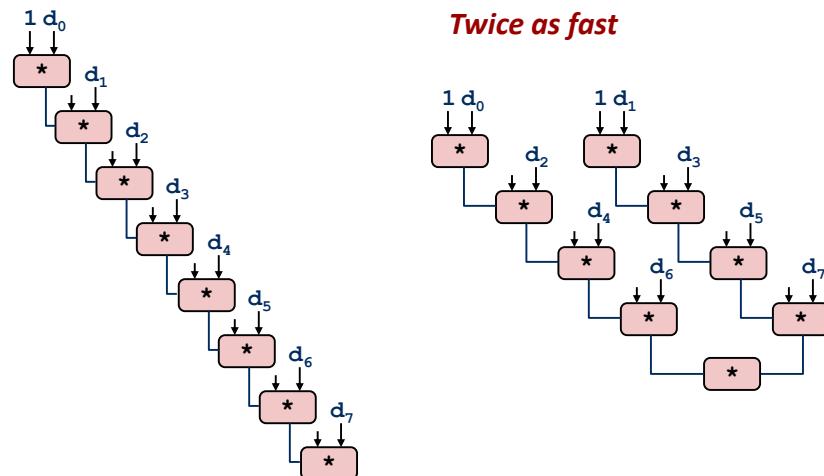
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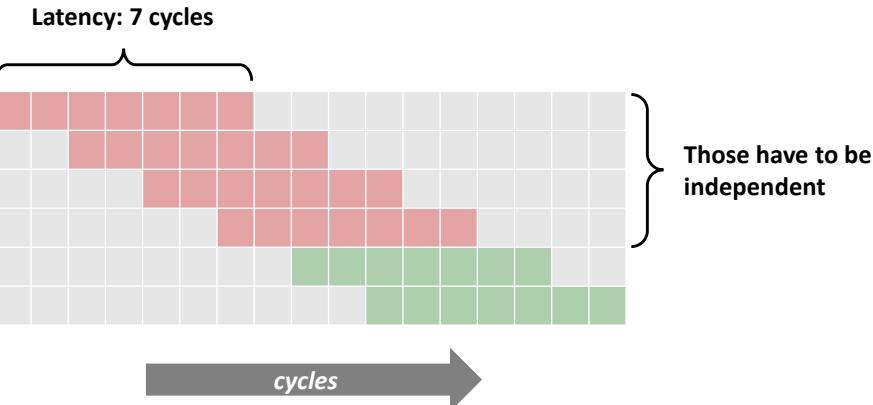
Last Time: ILP

- Latency/throughput (Pentium 4 fp mult: 7/2)



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Last Time: How Many Accumulators?



Based on this insight: $K = \#\text{accumulators} = \text{ceil}(\text{latency}/\text{cycles per issue})$

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Compiler Limitations

```
void combine4(vec_ptr v, data_t *dest)
{
    int i;
    int length = vec_length(v);
    data_t *d = get_vec_start(v);
    data_t t = IDENT;
    for (i = 0; i < length; i++)
        t = t OP d[i];
    *dest = t;
}
```



```
void unroll2_sa(vec_ptr v, data_t *dest)
{
    int length = vec_length(v);
    int limit = length-1;
    data_t *d = get_vec_start(v);
    data_t x0 = IDENT;
    data_t x1 = IDENT;
    int i;
    /* Combine 2 elements at a time */
    for (i = 0; i < limit; i+=2)
        x0 = x0 OP d[i];
        x1 = x1 OP d[i+1];
    /* Finish any remaining elements */
    for (; i < length; i++)
        x0 = x0 OP d[i];
    *dest = x0 OP x1;
}
```

- Associativity law does not hold for floats: illegal transformation
- No good way of handling choices (e.g., number of accumulators)
- *More examples of limitations today*

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Today

- **Measuring performance & benchmarking**

Section 3.2 in the tutorial

<http://spiral.ece.cmu.edu:8080/pub-spiral/abstract.jsp?id=100>

- **Optimizing compilers and optimization blockers**

- Overview
- Code motion
- Strength reduction
- Sharing of common subexpressions
- Removing unnecessary procedure calls
- Optimization blocker: Procedure calls
- Optimization blocker: Memory aliasing
- Summary

Chapter 5 in Computer Systems: A Programmer's Perspective, 2nd edition,

Randal E. Bryant and David R. O'Hallaron, Addison Wesley 2010

Part of these slides are adapted from the course associated with this book

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Benchmarking

- **First:** Validate/test your code!

- **Measure runtime (in [s] or [cycles]) for a set of relevant input sizes**

- seconds: actual runtime
- cycles: abstracts from CPU frequency

- **Usually: Compute and show performance (in [flop/s] or [flop/cycle])**

- **Careful:** Better performance ≠ better runtime (why?)

- Op count could differ
- Never show in one plot performance of two algorithms with substantially different op count

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How to Measure Runtime?

- **C clock()**
 - process specific, low resolution, very portable
- **gettimeofday**
 - measures wall clock time, higher resolution, somewhat portable
- **Performance counter (e.g., TSC on Intel)**
 - measures cycles (i.e., also wall clock time), highest resolution, not portable
- **Careful:**
 - measure only what you want to measure
 - ensure proper machine state
(e.g., cold or warm cache = input data is or is not in cache)
 - measure enough repetitions
 - check how reproducible; if not reproducible: fix it
- ***Getting proper measurements is not easy at all!***

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Problems with Timing

- **Too few iterations: inaccurate non-reproducible timing**
- **Too many iterations: system events interfere**
- **Machine is under load: produces side effects**
- **Multiple timings performed on the same machine**
- **Bad data alignment of input/output vectors:**
 - align to multiples of cache line (on Core: address is divisible by 64)
 - sometimes aligning to page boundaries (address divisible by 4096) makes sense
- **Machine was not rebooted for a long time: state of operating system causes problems**
- **Computation is input data dependent: choose representative input data**
- **Computation is inplace and data grows until an exception is triggered (computation is done with NaNs)**
- **You work on a computer that has dynamic frequency scaling (e.g., turbo boost)**
- ***Always check whether timings make sense, are reproducible***

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Benchmarks in Writing

- Specify experimental setup
 - platform
 - compiler and version
 - compiler flags used
- Plot: Very readable
 - Title, x-label, y-label should be there
 - Fonts large enough
 - Enough contrast (e.g., no yellow on white please)
 - Proper number format

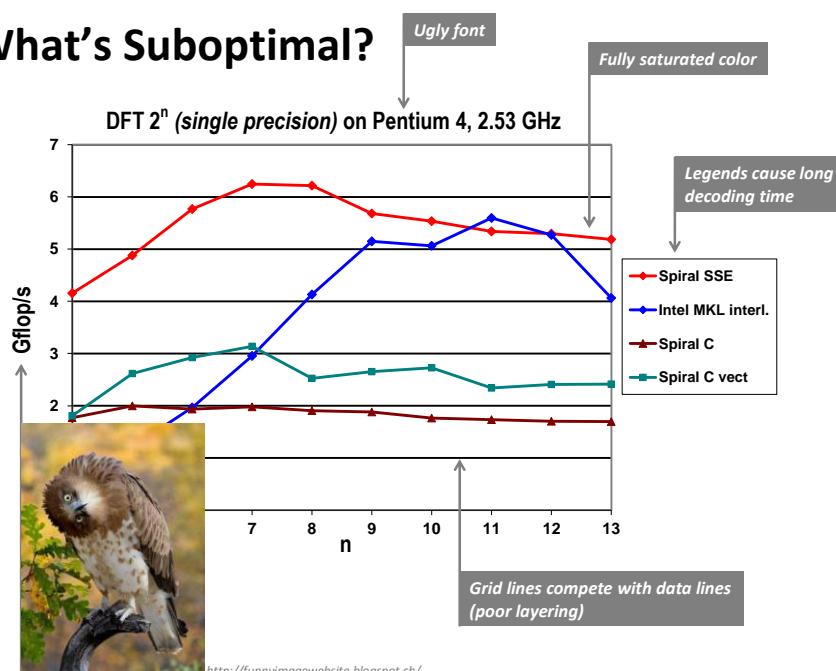
No: 13.254687; yes: 13.25

No: 2.0345e-05 s; yes: 20.3 μs

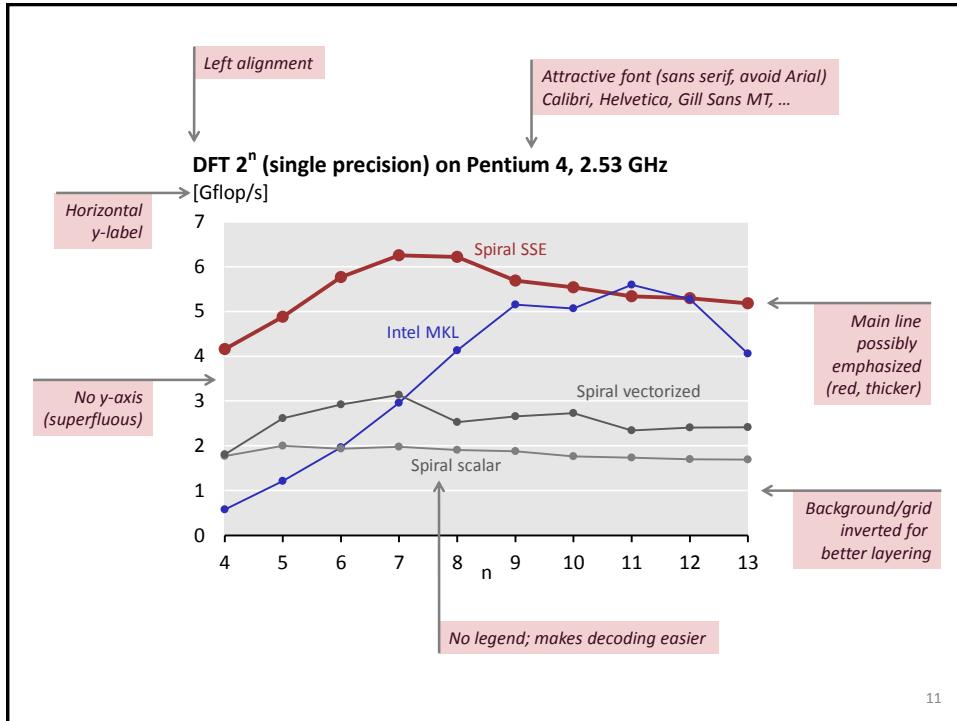
No: 100000 B; maybe: 100,000 B; yes: 100 KB

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What's Suboptimal?



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Today

- Measuring performance & benchmarking
- Optimizing compilers and optimization blockers
 - Overview
 - Removing unnecessary procedure calls
 - Code motion
 - Strength reduction
 - Sharing of common subexpressions
 - Optimization blocker: Procedure calls
 - Optimization blocker: Memory aliasing
 - Summary

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Optimizing Compilers



- Always use optimization flags:
 - gcc: *default is no optimization* (-O0)!
 - icc: some optimization is turned on
- Good choices for gcc/icc: -O2, -O3, -march=xxx, -mSSE3, -m64
 - Read in manual what they do
 - Try to understand the differences
- Try different flags and maybe different compilers

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Example (On Core 2 Duo)

```
double a[4][4];
double b[4][4];
double c[4][4];

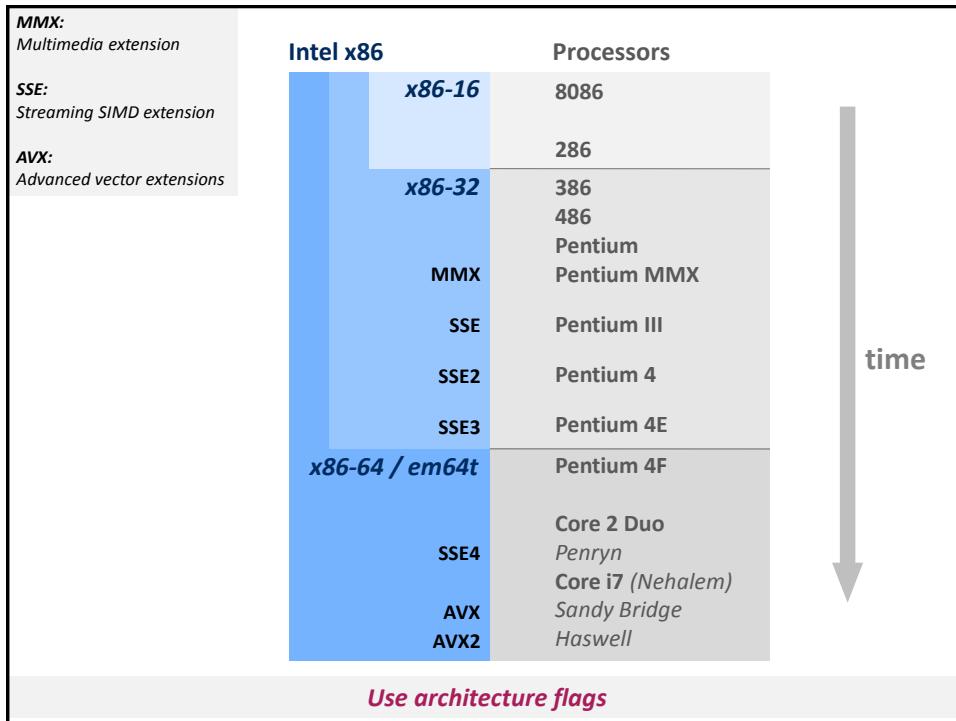
/* Multiply 4 x 4 matrices c = a*b + c */
void mmm(double *a, double *b, double *c) {
    int i, j, k;

    for (i = 0; i < 4; i++)
        for (j = 0; j < 4; j++)
            for (k = 0; k < 4; k++)
                c[i*4+j] += a[i*4 + k]*b[k*4 + j];
}
```

- Compiled without flags:
~1300 cycles
- Compiled with -O3 -m64 -march=... -fno-tree-vectorize
~150 cycles

Prevents use of SSE

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Optimizing Compilers

- Compilers are **good** at: mapping program to machine
 - register allocation
 - code selection and ordering (instruction scheduling)
 - dead code elimination
 - eliminating minor inefficiencies
- Compilers are **not good** at: algorithmic restructuring
 - for example to increase ILP, locality, etc.
 - cannot deal with choices
- Compilers are **not good** at: overcoming “optimization blockers”
 - potential memory aliasing
 - potential procedure side-effects

Limitations of Optimizing Compilers

- *If in doubt, the compiler is conservative*
- **Operate under fundamental constraints**
 - Must not change program behavior under any possible condition
 - Often prevents it from making optimizations when would only affect behavior under pathological conditions
- **Most analysis is performed only within procedures**
 - Whole-program analysis is too expensive in most cases
- **Most analysis is based only on *static* information**
 - Compiler has difficulty anticipating run-time inputs
 - Not good at evaluating or dealing with choices

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Organization

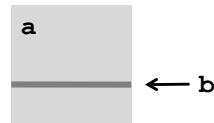
- Instruction level parallelism (ILP): an example
- **Optimizing compilers and optimization blockers**
 - Overview
 - *Code motion*
 - *Strength reduction*
 - *Sharing of common subexpressions*
 - Removing unnecessary procedure calls
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Code Motion

- Reduce frequency with which computation is performed
 - If it will always produce same result
 - Especially moving code out of loop (loop-invariant code motion)
- Sometimes also called precomputation

```
void set_row(double *a, double *b,
            int i, int n)
{
    int j;
    for (j = 0; j < n; j++)
        a[n*i+j] = b[j];
}
```



```
int j;
int ni = n*i;
for (j = 0; j < n; j++)
    a[ni+j] = b[j];
```

- Compiler is likely to do

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Strength Reduction

- Replace costly operation with simpler one
- Example: Shift/add instead of multiply or divide $16*x \rightarrow x \ll 4$
 - Utility machine dependent
- Example: Recognize sequence of products

```
for (i = 0; i < n; i++)
    for (j = 0; j < n; j++)
        a[n*i + j] = b[j];
```



```
int ni = 0;
for (i = 0; i < n; i++) {
    for (j = 0; j < n; j++)
        a[ni + j] = b[j];
    ni += n;
}
```

- Compiler is likely to do

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Share Common Subexpressions

- Reuse portions of expressions
- Compilers often not very sophisticated in exploiting arithmetic properties

3 mults: $i*n, (i-1)*n, (i+1)*n$

```
/* Sum neighbors of i,j */
up    = val[(i-1)*n + j];
down  = val[(i+1)*n + j];
left   = val[i*n      + j-1];
right  = val[i*n      + j+1];
sum    = up + down + left + right;
```

1 mult: $i*n$

```
int inj = i*n + j;
up     = val[inj - n];
down   = val[inj + n];
left   = val[inj - 1];
right  = val[inj + 1];
sum    = up + down + left + right;
```

- In simple cases compiler is likely to do

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Organization

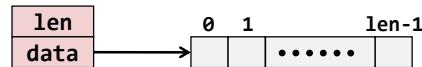
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Compiler is likely to do

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Example: Data Type for Vectors

```
/* data structure for vectors */
typedef struct{
    int len;
    double *data;
} vec;
```



```
/* retrieve vector element and store at val */
int get_vec_element(vec *v, int idx, double *val)
{
    if (idx < 0 || idx >= v->len)
        return 0;
    *val = v->data[idx];
    return 1;
}
```

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Example: Summing Vector Elements

```
/* retrieve vector element and store at val */
int get_vec_element(vec *v, int idx, double *val)
{
    if (idx < 0 || idx >= v->len)
        return 0;
    *val = v->data[idx];
    return 1;
}
```

```
/* sum elements of vector */
double sum_elements(vec *v, double *res)
{
    int i;
    n = vec_length(v);
    *res = 0.0;
    double t;

    for (i = 0; i < n; i++) {
        get_vec_element(v, i, &t);
        *res += t;
    }
    return res;
}
```

Overhead for every fp +:

- One fct call
- One <
- One \geq
- One $\mid\mid$
- One memory variable access

Potential big performance loss

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Removing Procedure Call

```
/* sum elements of vector */
double sum_elements(vec *v, double *res)
{
    int i;
    n = vec_length(v);
    *res = 0.0;
    double t;

    for (i = 0; i < n; i++) {
        get_vec_element(v, i, &t);
        *res += t;
    }
    return res;
}

/* sum elements of vector */
double sum_elements_opt(vec *v, double *res)
{
    int i;
    n = vec_length(v);
    *res = 0.0;
    double *data = get_vec_start(v);

    for (i = 0; i < n; i++)
        *res += data[i];
    return res;
}
```

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Removing Procedure Calls

- Procedure calls can be very expensive
- Bound checking can be very expensive
- Abstract data types can easily lead to inefficiencies
 - Usually avoided in superfast numerical library functions
- *Watch your innermost loop!*
- *Get a feel for overhead versus actual computation being performed*

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Further Inspection of the Example

```
vector.c // vector data type      Intel Xeon E3-1285L v3 (Haswell)  
sum.c    // sum                  CC=gcc -w -O3 -std=c99 -march=core-avx2  
sum_opt.c // optimized sum     Intel Atom D2550  
main.c   // timing              CC=gcc -w -std=c99 -O3 -march=atom
```

```
$(CC) -c -o vector.o vector.c      Xeon: 7.2 cycles/add  
$(CC) -c -o sum.o sum.c           Atom: 28 cycles/add  
$(CC) -c -o main.o main.c  
$(CC) -o vector vector.o sum.o main.o
```

```
$(CC) -c -o vector.o vector.c      Xeon: 2.4 cycles/add  
$(CC) -c -o sum_opt.o sum_opt.c   Atom: 6 cycles/add  
$(CC) -c -o main.o main.c  
$(CC) -o vector vector.o sum_opt.o main.o
```

```
$(CC) -c -o vector.o vector.c sum.c Xeon: 2.4 cycles/add  
$(CC) -c -o main.o main.c          Atom: 6 cycles/add  
$(CC) -o vector vector.o main.o
```

What's happening here?

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Function Inlining

- Compilers may be able to do function inlining
 - Replace function call with body of function
 - Usually requires that source code is compiled together

```
/* retrieve vector element and store at val */  
int get_vec_element(vec *v, int idx, double *val)  
{  
    if (idx < 0 || idx >= v->len)  
        return 0;  
    *val = v->data[idx];  
    return 1;  
}  
  
/* sum elements of vector */  
double sum_elements(vec *v, double *res)  
{  
    int i;  
    n = vec_length(v);  
    *res = 0.0;  
    double t;  
  
    for (i = 0; i < n; i++) {  
        get_vec_element(v, i, &t);  
        *res += t;  
    }  
    return res;  
}
```

- Enables other optimizations
- *Problem:* performance libraries distributed as binary

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 - Optimization blocker: Memory aliasing
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Compiler is likely to do

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Optimization Blocker #1: Procedure Calls

- Procedure to convert string to lower case

```
void lower(char *s)
{
    int i;
    for (i = 0; i < strlen(s); i++)
        if (s[i] >= 'A' && s[i] <= 'Z')
            s[i] -= ('A' - 'a');
}
```

O(n²) instead of O(n)

```
/* My version of strlen */
size_t strlen(const char *s)
{
    size_t length = 0;
    while (*s != '\0') {
        s++;
        length++;
    }
    return length;
}
```

O(n)

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Improving Performance

```
void lower(char *s)
{
    int i;
    for (i = 0; i < strlen(s); i++)
        if (s[i] >= 'A' && s[i] <= 'Z')
            s[i] -= ('A' - 'a');
}
```

```
void lower(char *s)
{
    int i;
    int len = strlen(s);
    for (i = 0; i < len; i++)
        if (s[i] >= 'A' && s[i] <= 'Z')
            s[i] -= ('A' - 'a');
}
```

- Move call to `strlen` outside of loop
- Form of code motion/precomputation

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Optimization Blocker: Procedure Calls

- Why couldn't compiler move `strlen` out of inner loop?
 - Procedure may have side effects
- *Compiler usually treats procedure call as a black box that cannot be analyzed*
 - Consequence: conservative in optimizations
- In this case the compiler may actually do it if `strlen` is recognized as built-in function whose properties are known

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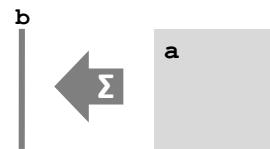
Compiler is likely to do

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Optimization Blocker: Memory Aliasing

```
/* Sums rows of n x n matrix a
   and stores in vector b */
void sum_rows1(double *a, double *b, int n) {
    int i, j;

    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i*n + j];
    }
}
```



- Code updates $b[i]$ (= memory access) on every iteration

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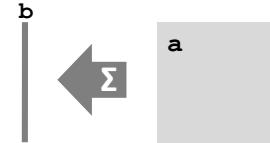
Optimization Blocker: Memory Aliasing

```
/* Sums rows of n x n matrix a
   and stores in vector b */
void sum_rows1(double *a, double *b, int n) {
    int i, j;

    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i*n + j];
    }
}
```

```
/* Sums rows of n x n matrix a
   and stores in vector b */
void sum_rows2(double *a, double *b, int n) {
    int i, j;

    for (i = 0; i < n; i++) {
        double val = 0;
        for (j = 0; j < n; j++)
            val += a[i*n + j];
        b[i] = val;
    }
}
```



Does compiler optimize this?

No!

Why?

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Reason: Possible Memory Aliasing

- If memory is accessed, compiler assumes the possibility of side effects
- Example:

```
/* Sums rows of n x n matrix a
   and stores in vector b */
void sum_rows1(double *a, double *b, int n) {
    int i, j;

    for (i = 0; i < n; i++) {
        b[i] = 0;
        for (j = 0; j < n; j++)
            b[i] += a[i*n + j];
    }
}
```

```
double A[9] =
{ 0, 1, 2,
  4, 8, 16,
  32, 64, 128};

double B[3] = A+3;

sum_rows1(A, B, 3);
```

Value of B:

init: [4, 8, 16]
i = 0: [3, 8, 16]
i = 1: [3, 22, 16]
i = 2: [3, 22, 224]

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Removing Aliasing

```
/* Sums rows of n x n matrix a
   and stores in vector b */
void sum_rows2(double *a, double *b, int n) {
    int i, j;

    for (i = 0; i < n; i++) {
        double val = 0;
        for (j = 0; j < n; j++)
            val += a[i*n + j];
        b[i] = val;
    }
}
```

- **Scalar replacement:**

- Copy array elements *that are reused* into temporary variables
- Perform computation on those variables
- Enables register allocation and instruction scheduling
- Assumes no memory aliasing (otherwise incorrect)

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Optimization Blocker: Memory Aliasing

- **Memory aliasing:**

Two different memory references write to the same location

- **Easy to have happen in C**

- Since allowed to do address arithmetic
- Direct access to storage structures

- **Hard to analyze = compiler cannot figure it out**

- Hence is conservative

- **Solution: Scalar replacement in innermost loop**

- Copy memory variables that are reused into local variables
- Basic scheme:

Load: $t1 = a[i], t2 = b[i+1], \dots$

Compute: $t4 = t1 * t2, \dots$

Store: $a[i] = t12, b[i+1] = t7, \dots$

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Example: MMM

Which array elements are reused? *All of them! But how to take advantage?*

```
void mmm(double const * A, double const * B, double * C, size_t N) {
    for( size_t k6 = 0; k6 < N; k6++ )
        for( size_t i5 = 0; i5 < N; i5++ )
            for( size_t j7 = 0; j7 < N; j7++ )
                C[N*i5 + j7] = C[N*i5 + j7] + A[N*i5 + k6] * B[j7 + N*k6]; }
```

tile each loop (= blocking MMM)

```
void mmm(double const * A, double const * B, double * C, size_t N) {
    for( size_t i21 = 0; i21 < N; i21+=2 )
        for( size_t j23 = 0; j23 < N; j23+=2 )
            for( size_t k22 = 0; k22 < N; k22+=2 )
                for( size_t kk25 = 0; kk25 < 2; kk25++ )
                    for( size_t ii24 = 0; ii24 < 2; ii24++ )
                        for( size_t jj26 = 0; jj26 < 2; jj26++ )
                            C[N*i21 + N*ii24 + j23 + jj26] = C[N*i21 + N*ii24 + j23 + jj26] +
                                A[N*i21 + N*ii24 + k22 + kk25] * B[j23 + jj26 + N*k22 + N*kk25]; }
```

unroll inner three loops

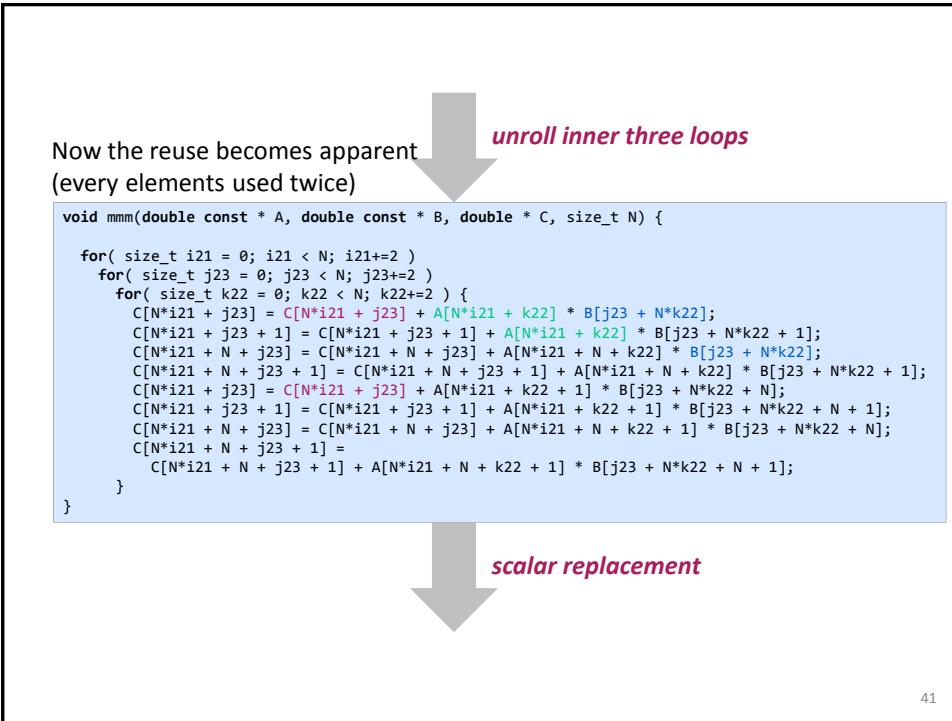
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Now the reuse becomes apparent
(every elements used twice)

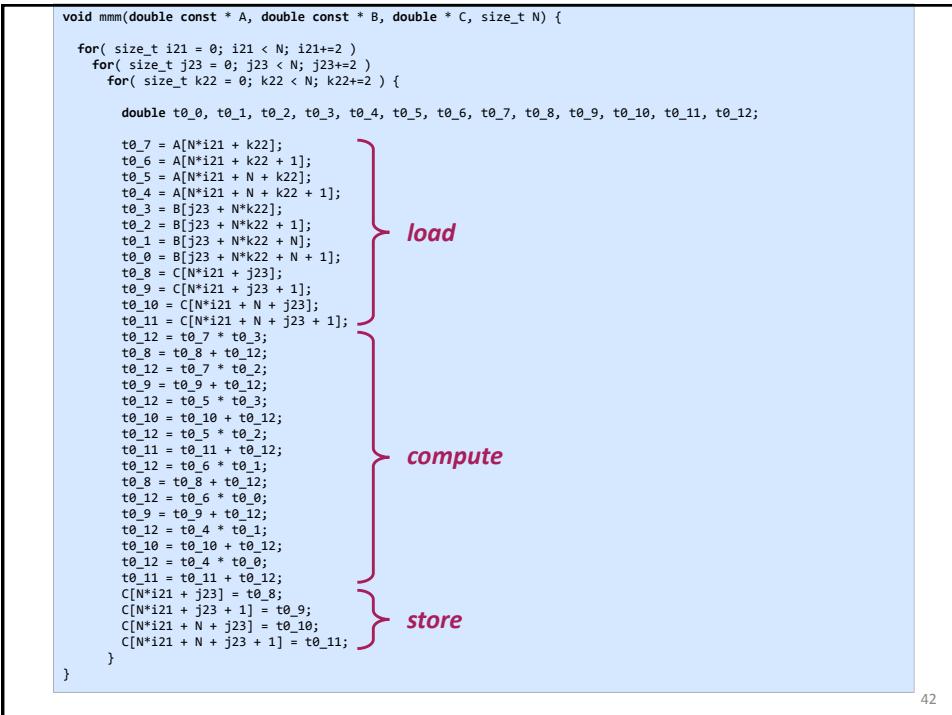
unroll inner three loops

```
void mmm(double const * A, double const * B, double * C, size_t N) {
    for( size_t i21 = 0; i21 < N; i21+=2 )
        for( size_t j23 = 0; j23 < N; j23+=2 )
            for( size_t k22 = 0; k22 < N; k22+=2 ) {
                C[N*i21 + j23] = C[N*i21 + j23] + A[N*i21 + k22] * B[j23 + N*k22];
                C[N*i21 + j23 + 1] = C[N*i21 + j23 + 1] + A[N*i21 + k22] * B[j23 + N*k22 + 1];
                C[N*i21 + N + j23] = C[N*i21 + N + j23] + A[N*i21 + N + k22] * B[j23 + N*k22];
                C[N*i21 + N + j23 + 1] = C[N*i21 + N + j23 + 1] + A[N*i21 + N + k22] * B[j23 + N*k22 + 1];
                C[N*i21 + j23] = C[N*i21 + j23] + A[N*i21 + k22 + 1] * B[j23 + N*k22 + N];
                C[N*i21 + j23 + 1] = C[N*i21 + j23 + 1] + A[N*i21 + k22 + 1] * B[j23 + N*k22 + N + 1];
                C[N*i21 + N + j23] = C[N*i21 + N + j23] + A[N*i21 + N + k22 + 1] * B[j23 + N*k22 + N];
                C[N*i21 + N + j23 + 1] =
                    C[N*i21 + N + j23 + 1] + A[N*i21 + N + k22 + 1] * B[j23 + N*k22 + N + 1];
            }
}
```

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Effect on Runtime?

Intel Core i7-2600 (Sandy Bridge)
compiler: `icc 12.1`
flags: `-O3 -no-vec -no-ipo -no-ip`

	N = 4	N = 100
Triple loop	202	2.3M

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Effect on Runtime?

Intel Core i7-2600 (Sandy Bridge)
compiler: `icc 12.1`
flags: `-O3 -no-vec -no-ipo -no-ip`

	N = 4	N = 100
Triple loop	202	2.3M
Six-fold loop	144	2.3M
+ Inner three unrolled	166	2.4M
+ scalar replacement	106	1.6M

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Can Compiler Remove Aliasing?

```
for (i = 0; i < n; i++)
    a[i] = a[i] + b[i];
```

Potential aliasing: Can compiler do something about it?

Compiler can insert runtime check:

```
if (a + n < b || b + n < a)
    /* further optimizations may be possible now */
    ...
else
    /* aliased case */
    ...
```

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Removing Aliasing With Compiler

- **Globally with compiler flag:**

- -fno-alias, /Oa
- -fargument-noalias, /Qalias-args- (function arguments only)

- **For one loop: pragma**

```
void add(float *a, float *b, int n) {
    #pragma ivdep
    for (i = 0; i < n; i++)
        a[i] = a[i] + b[i];
}
```

- **For specific arrays: restrict (needs compiler flag -restrict, /Qrestrict)**

```
void add(float *restrict a, float *restrict b, int n) {
    for (i = 0; i < n; i++)
        a[i] = a[i] + b[i];
}
```

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Organization

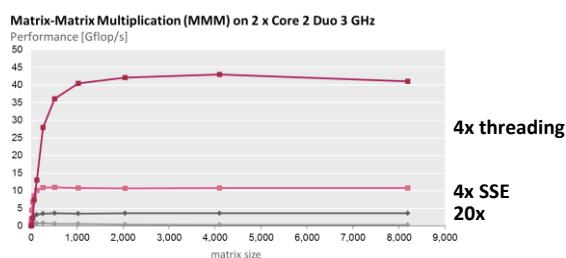
- Instruction level parallelism (ILP): an example
- Optimizing compilers and optimization blockers
 - Overview
 - Removing unnecessary procedure calls
 - Code motion
 - Strength reduction
 - Sharing of common subexpressions
 - Removing unnecessary procedure calls
 - Optimization blocker: Procedure calls
 - Optimization blocker: Memory aliasing
 - **Summary**

Compiler is likely to do

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Summary

- One can easily loose 10x, 100x in runtime or even more



- What matters besides operation count:
 - Code style (unnecessary procedure calls, unrolling + ..., reordering, ...)
 - Algorithm structure (instruction level parallelism, locality, ...)
 - Data representation (complicated structs or simple arrays)

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Summary: Optimize at Multiple Levels

- **Algorithm:**
 - Evaluate different algorithm choices
 - Restructuring may be needed (ILP, locality)
- **Data representations:**
 - Careful with overhead of complicated data types
 - Best are arrays
- **Procedures:**
 - Careful with overhead
 - They are black boxes for the compiler
- **Loops:**
 - Often need to be restructured (ILP, locality)
 - Unrolling often necessary *to enable other optimizations*
 - Watch the innermost loop bodies

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Numerical Functions

- **Use arrays (simple data structure) if possible**
- **Unroll to some extent**
 - To make ILP explicit
 - To enable scalar replacement and hence register allocation for variables that are reused

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